WORKSHOP "AFFINE SPACES, ALGEBRAIC GROUP ACTIONS, AND LNDS"

The talks take place at the A.N. Kolmogorov Bhavan, ISI Kolkata, either at the L-infinity (5th Floor) or the HGU Conference Room (2nd Floor).

Flexible varieties and images of affine spaces

Ivan Arzhantsev
HSE University

13.03.2023, 10:30 AM at L-infinity

We study the problem which algebraic varieties may be obtained as images of affine spaces under a morphism of algebraic varieties. More precisely, we prove that every non-degenerate toric variety, every homogeneous space of a connected linear algebraic group without non-constant invertible regular functions, and every variety covered by affine spaces admits a surjective morphism from an affine space. These results are based on the concept of a flexible quasi-affine variety and the quotient presentation coming from the theory of Cox rings. We plan to survey basic facts on flexible varieties and give many examples of varieties with a multiply transitive action of the automorphism group.

On Affine Fibrations

Amartya Kumar Dutta
ISI Kolkata

13.03.2023, 12:10 PM at L-infinity

In this survey talk on Affine Fibrations, I'll mention some of the results, examples and open problems in the area, highlighting their impact on other challenging problems on affine spaces, especially the Epimorphism Problems and the Zariski Cancellation Problem.

Root subgroups on spherical varieties

Roman Avdeev
HSE University

13.03.2023, 2:30 PM at L-infinity

In the study of automorphism groups of toric varieties, a key role is played by one-parameter additive groups normalized by the acting torus. Such subgroups are called root subgroups and each of them is uniquely determined by its weight, called a Demazure root of the corresponding toric variety. Moreover, the set of all Demazure roots admits an explicit combinatorial description in terms of the fan defining the toric variety, and this description is especially simple in the case where the variety is affine.

In the setting of arbitrary connected reductive groups acting on algebraic varieties, a natural generalization of toric varieties is given by spherical varieties. A spherical variety is an algebraic variety X equipped with an action of a connected reductive group G in such a way that a Borel subgroup B of G has a dense open orbit in X. A proper generalization of root subgroups for spherical varieties is given by one-parameter additive groups normalized by B, which are called B-root subgroups. In the talk we shall discuss B-root subgroups on spherical varieties, including basic properties, applications, and open problems.

The talk is based on joint works of the speaker with I. Arzhantsev and with V. Zhgoon.

Nested automorphism groups

Aleksandr Perepechko
HSE University

14.03.2023, 10:30 AM at HGU Conference Room

A closed subgroup of an ind-group is called *nested* if it equals the union of its algebraic subgroups. For an affine algebraic variety X, we study nested subgroups of the automorphism group Aut(X).

An affine variety X is called *rigid* if the actions of $\mathbb{G}_a$ on X are absent, and *semirigid*, if they are equivalent to each other (in particular, commute). In joint works with Zaidenberg and Regeta we conjectured that the neutral component of Aut(X) for a rigid or semirigid X is nested, and so equals a semidirect product of an algebraic torus and an abelian unipotent subgroup. We prove in this case that the algebraically generated subgroup of the neutral component is nested.

We also survey recent results by Kraft and Zaidenberg on the structure of nested unipotent subgroups with an open orbit in X.

Residual Coordinates of Affine Fibrations and their applications

Prosenjit Das
IIST Thiruvananthapuram

14.03.2023, 12:10 PM at HGU Conference Room

The concept of the residual coordinate of a polynomial ring in two variables was introduced by Bhatwadekar in 1988. Subsequently, Bhatwadekar-Dutta, in 1993, established the theory of residual coordinates of polynomial algebras, which has been an important tool for solving many problems in affine algebraic geometry. Later, during 2013–2014, Kahoui–Ouali and Das–Dutta introduced the concept and the theory of residual coordinates of affine fibrations to tackle problems related to affine fibrations. This talk shall survey the theory of residual coordinates and their applications in solving different problems in affine algebraic geometry.

Flexible varieties and images of affine spaces II

Ivan Arzhantsev
HSE University

14.03.2023, 2:30 PM at HGU Conference Room

We continue the talk on flexible varieties and images of affine spaces.

Modified Makar-Limanov invariant

Sergei Gaifullin
HSE University

15.03.2023, 10:30 AM at HGU Conference Room

This talk is based on a joint work with Anton Shafarevich. The Makar-Limanov invariant $ML(X)$ of an affine variety X is the intersection of all kernels of locally nilpotent derivations on $\mathbb{K}[X]$. The Derksen invariant $HD(X)$ is the subalgebra in $\mathbb{K}[X]$ generated by all kernels of locally nilpotent derivations.

We investigate modified Makar-Limanov and modified Derksen invariants of an affine algebraic variety. The modified Makar-Limanov invariant $ML^*(X)$ is the intersection of kernels of all locally nilpotent derivations with slices and the modified Derksen invariant $HD^*(X)$ is the subalgebra generated by these kernels. We prove that for every variety $ML(X) = ML^*(X)$, if there exists a locally nilpotent derivation with a slice. Also we construct an example of a variety admitting a locally nilpotent derivation with a slice such that $HD^*(X) \neq HD(X)$.

Graded automorphisms of the algebra of polynomials in three variables

Anton Trushin
HSE University

15.03.2023, 12:10 PM at HGU Conference Room

In 2004 Shestakov and Umirbaev proved that the Nagata automorphism of the polynomial algebra in three variables is wild. We fix a $\mathbb{Z}$-grading on this algebra and consider graded-wild automorphisms, i.e. such automorphisms that can not be decomposed onto elementary automorphisms respecting the grading. We describe all gradings allowing graded-wild automorphisms and construct a system of group-generating automorphisms that preserve a given grading.

On Generalised Danielewski and Asanuma Varieties

Parnashree Ghosh
ISI Kolkata

15.03.2023, 2:30 PM at HGU Conference Room

Let k be a field, m a positive integer, $\mathbb{V}$ an affine subvariety of $\mathbb{A}^{m+3}$ defined by a linear relation of the form $x_1^{r_1} \cdots x_m^{r_m} y = F(x_1, \ldots, x_m, z, t)$, A the coordinate ring of $\mathbb{V}$ and $G = X_1^{r_1} \cdots X_m^{r_m} Y - F(X_1, \ldots, X_m, Z, T)$. We name the varieties as "Generalised Asanuma varieties".

In the first part of this talk, we exhibit several necessary and sufficient conditions for $\mathbb{V}$ to be isomorphic $\mathbb{A}^{m+2}$ and G to be a coordinate in $k[X1, ..., Xm, Y, Z, T]$, under a certain hypothesis on F. Our main result immediately yields a family of higher-dimensional linear hyperplanes for which the Abhyankar-Sathaye Conjecture holds.

In the second part, we show an extension of a result of Dubouloz on the Cancellation Problem in higher dimensions ($\geq$2) over the field of complex numbers to fields of arbitrary characteristic. We then apply the generalised result to describe the Makar-Limanov and Derksen invariant of generalised Asanuma varieties under certain hypotheses.

This talk is based on joint works with Neena Gupta.

Automorphism group of toral varieties

Anton Shafarevich
HSE University

16.03.2023, 10:30 AM at L-infinity

The talk is based on the joint work with A. Trushin.

Let $\mathbb{K}$ – be an algebraically closed field of characteristic zero and let X — be an affine algebraic variety over $\mathbb{K}$. A variety X is called *toral* if X is isomorphic to a subvariety in the algebraic torus $T = (\mathbb{K}^*)^n$.

Toral varieties are rigid, that is, they do not admit nontrivial actions of the additive group of the field $\mathbb{K}$. It was shown by Arzhantsev and Gaifullin that the automorphism group of a rigid variety contains a unique maximal torus. Also there were given examples of rigid varieties for which it was possible to find a group of regular automorphisms.

In my talk, we will discuss about some properties of the automorphism group of toral varieties.

Theorem. Let X be a toral variety and T a maximal torus in the automorphism group. Then the variety X is isomorphic to the direct product $Y \times T$, where Y is a toral variety with a discrete automorphism group. Moreover

$$\mathrm{Aut}(X) = \mathrm{Aut}(Y) \leftthreetimes (\mathrm{GL}_k(\mathbb{Z}) \leftthreetimes (\mathbb{K}[Y]^*)^k),$$

where k is the dimension of the torus T and $\mathbb{K}[Y]^*$ is the multiplicative group of invertible regular functions on Y.

It is known that if X is an irreducible algebraic variety, then the factor group $\mathbb{K}[X]^*/\mathbb{K}^*$ is a free finitely generated abelian group. If X is a toral variety, then the rank of $\mathbb{K}[X]^*/\mathbb{K}^*$ is at least the dimension of X. We will discuss how to find the automorphism group of toral varieties X whose rank $\mathbb{K}[X]^*/\mathbb{K}^*$ is equal to or differs by one from the dimension of X.

Finite generation of kernel of locally nilpotent derivations of $R^{[n]}$

Swapnil Lokhande
IIIT Vadodara

16.03.2023, 12:10 PM at L-infinity

Let R be an integral domain containing $\mathbb{Q}$, $B = R[n]$ a polynomial ring in n variables over R and D a locally nilpotent derivation of B over R. If R is a field and $n < 4$ then kernel of D is finitely generated R-algebra. It is also finitely generated in the case R UFD, $n < 3$ and R dedekind domain, $n = 3$. Kernel of D need not be a finitely generated R-algebra, in the case of R UFD, $n > 2$ or R a field, $n > 4$. In this talk we will see the finite generation of kernel is D in case of $n = 3$ and D a triangular monomial derivation. We will also see the generalization of the Daigle Freudenburg counterexample for the case k field and $n > 4$.

On isotropy subgroups

Nikhilesh Dasgupta
NMIMS Mumbai

16.03.2023, 2:30 PM at L-infinity

Let k be an algebraically closed field of characteristic zero. For an affine k-domain B and a locally nilpotent derivation D on B, the isotropy subgroup of D, denoted by $\mathrm{Aut}(B)_D$ is defined to be $\{\phi \in \mathrm{Aut}(B) \mid \phi D = D\phi\}$. In this talk, firstly I shall describe the structure of the isotropy subgroup for the canonincal locally nilpotent derivation and its replicas for certain almost rigid domains. Secondly, I shall also discuss properties of isotropy subgroups of locally nilpotent derivations on $k^{[2]}$ and $k^{[3]}$.

This talk is based on a joint work with Animesh Lahiri and also on an ongoing one with Sergey Gayfullin.

Affine monoids of corank 1

Yulia Zaitseva
HSE University

17.03.2023, 10:30 AM at HGU Conference Room

An affine algebraic monoid is an irreducible affine algebraic variety X with an associative multiplication which is a morphism of algebraic varieties and has a unity. The group of invertible elements of an algebraic monoid X is an algebraic group, Zariski open in X. A monoid is called commutative (reductive, of rank r, ...) if the group of invertible elements is commutative (reductive, of rank r, ...), respectively. I plan to talk about the current progress in classification of noncommutative affine monoids of corank one.

Image ideals of nice and quasi-nice derivations over a UFD

Animesh Lahiri
Chennai Mathematical Institute

17.03.2023, 12:10 PM at HGU Conference Room

In this talk, for a finitely generated $\mathbb{Q}$-algebra R, we will give a set of generators for each of the image ideals of irreducible nice and quasi-nice R-derivations on the polynomial ring $R[X, Y]$, where R is a UFD. Moreover, when R is a PID, we will show similar results can be obtained for such derivations on the polynomial ring $R[X, Y, Z]$. The talk is based on my recent work with Nikhilesh Dasgupta.

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